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G.R.I.M. (Departimento di Matematica e Informatica, University of Palermo, Italy)

Functions of operations and operands in school mathematics and physics: a complex interdisciplinary (de)mathematised phenomenology

Andreas Moutsios-Rentzos, Georgios Kritikos & Fragkiskos Kalavasis
Department of Pre-School Education Sciences and Educational Design, University of the Aegean, Dimokratias Av. 1, 85100, Rhodes, Greece.

E-mail: amoutsiosrentzos@aegean.gr

Abstract. In this laboratory of interdisciplinary reflection, we consider the school unit as an open learning organisation. This systemic view allows us to observe the implicit processes of (de)mathematisation that take place during the teaching-learning of school mathematics and physics through common to both mathematical formulae. The participants are invited to reflect upon specific examples with operations and operands in mathematics and physics, to realise the diverse implicit intentionalities that give meaning to cognitive reflexes and conventions that remain hidden for the learners and for the teachers of different disciplines. The present school reality promotes the compartmentalisation of knowledge, hindering the identification of both the specific to each discipline and the invariant across disciplines meanings. Such invisible obstacles may become visible, thus engineerable, within a reformed model of school that facilitates the emergence of the complex interdisciplinary teaching-learning space amongst and within school mathematics and physics.

Résumé. Dans ce laboratoire de réflexion interdisciplinaire, nous considérons l'unité scolaire comme une organisation d'apprentissage ouverte. Cette vision systémique nous permet d'observer les processus implicites de la (de)mathématisation qui se déroulent pendant l'enseignement et l'apprentissage des mathématiques et de la physique à l'école par des formules mathématiques communs. Les participants sont invités à réfléchir à des exemples spécifiques avec des opérations et des opérandes en mathématiques et en physique, pour réaliser les diverses intentionnalités implicites qui donnent un sens aux réflexes cognitifs et aux conventions qui restent cachés pour les étudiants et les enseignants des différentes disciplines. La réalité actuelle de l'école favorise la compartimentation des connaissances, ce qui entrave l'identification à la fois de la discipline spécifique et des invariants á travers des disciplines. De tels obstacles invisibles peuvent devenir visibles, donc gérable, dans un modèle d'école réformé qui facilite l'émergence de l'espace interdisciplinaire d'enseignement et d'apprentissage entre les mathématiques scolaires et la physique scolaire.

1. The school unit learning system: emerging interdisciplinary realities

These Mathematics and physics lie at the heart of the contemporary curricula, reflecting their crucial role in the contemporary societies. In particular, the ability to identify, link and apply scientific and mathematical models within the broader virtual-sociocultural practices appear to be an internationally acknowledged critical issue of education. Within this complex reality, in this laboratory, we view the school unit as a system to investigate the inter-acting and inter-related constructions that affect both mathematics and physics education within the complex and multi-levelled labour of the school unit (Bertalanfy, 1968; Davis & Simmt, 2003). The school unit functions as an intelligent, learning organisation with members of diverse roles, relationships and intentionalities transforming and being transformed by the organisation itself. Learning is conceptualised as linking links (Moutsios-Rentzos & Kalavasis, 2016). In the school unit, learning involves the transformation of the system, as the links transform the produced meaning, thus resulting to a new state of equilibrium. The systemic reflections upon experiences and/or actions identify change, whilst through their communication the new learning emerges at the individual and collective foreground. Such processes and states may be scarcely explicit within the same school course, whilst they seem to be completely missing across courses of different disciplines. It is argued that a school unit needs to explicitly address these phenomena to facilitate the students appropriately linking links within and across courses.

In this Reflective Interdisciplinary Laboratory, we focus on the (de)mathematisation processes (Gellert & Jablonka, 2007) that co-exist within and across school mathematics and physics, positing that appropriate *interdisciplinary reflective practices* may facilitate the construction of both intra-/inter-disciplinary meanings and learning. Through interdisciplinary individual and collective, descriptive, comparative and critical reflections (Jay & Johnson, 2002; Nissilä, 2005), common practices may emerge that would allow linking mathematics and physics, whilst clearly differentiating each discipline, thus obtaining internally-referenced (for the learners) meaning, rather than remaining an externally imposed differentiation. The co-development of the new qualities of the teaching-learning interdisciplinary experiences is expected to appropriately transform the learning abilities of the school unit, in line with the contemporary educational and social requirements and expectations.

2. Operations and operands in school mathematics and physics

Number operations lie at the heart of mathematics; from the early years, till university mathematics. In school mathematics, number operations work with three main types of operands (numerical, symbolic, geometrical), whilst magnitudes may be employed as a context (for example, word problems; Verschaffel, Greer & De Corte, 2000). Though the context may seem to be epistemologically irrelevant to mathematics, it lies at the heart of physics, giving physical meaning to the mathematical formulae, notably operations. Moreover, context seems to be crucial for mathematics education as, for example, through horizontal and vertical mathematisations, the students may be guided to meaningfully re-invent a mathematical idea (Gravemeijer, 1994).

The workshop simulates an interdisciplinary re-visit of the meaning of operations and operands in school mathematics and physics, organised in three parts. At the crux of the workshop lies the collective, comparative reflections upon signs and expressions of operations and operands that are used both in mathematics and physics, in order for the participants to experience that this common use in both sciences use indicates the historical and epistemological dialectic within each discipline and, importantly, between the two disciplines. First, we shall reflect upon operations and operands within mathematics. Drawing upon the ontological differences amongst algebraic and geometrical object (Duval, 2006), the algebraic operations in the expression of geometrical relationships may conceal the complexity of the geometrical meanings (Moutsios-Rentzos, Spyrou & Peteinara, 2014). For example, in an indicative activity of the first phase, we shall consider the algebraic expression of the Pythagorean Theorem. The algebraic addition of "a^2+b^2" is broader than its restriction to the set of positive real numbers; a restriction enforced by the geometrical nature of the operands. The participants will reflect upon the geometrical meaning of "a^2+b^2". For example: Does it refer to the addition of areas and, in that case, in which way does the relationship of areas characterise the angle of a triangle and what is the geometrical meaning of addition? Does it refer to the addition of two divisions the sum of which is constant ("$a^2/c^2+b^2/c^2=1$")? In that case, in which way does the ratio relationship characterise the angle of a triangle? And, what is the geometrical meaning of division?

In the second part, we shall consider the transformations of meanings that occur in the operations when the operand is a physical notion. For example, in an indicative activity of the first phase, we consider the definition of electric capacitance, as found in the textbook of the second grade of the Greek Lyceum (age 16 years old), is: "Capacitance C of a capacitor is the scalar physical quantity, which is equal to the quotient of the electric charge Q of the capacitor over the electric potential V of the capacitor. $C=Q/V$" (Alexakis et al., 2013, p. 32). The participants will be asked to calculate the capacitance of a given capacitor when the electric potential is doubled. Given the fact that the definition uses the word "quotient" and drawing upon the operation of division as a procedure, the participants may infer that the capacitance is halved. Nevertheless, it is argued that the nature of the operand, the physical meaning of capacitance (that is, the fact that the capacitance of a given object is a characteristic of the capacitor and as such remains constant), gives meaning to the performed operations, clarifying the nature of the relationship and in specific that 'C' is a constant, whilst Q, V are variables and, thus, the relationship is of the form "$y=ax$" i.e. an analogy. Following these, a more mathematically and physically compatible definition may be chosen; for example, the *physical notion* of capacitance may be defined as *the constant ratio* of the held charge over the applied electric potential. Drawing upon the *mathematical* notion of ratio, the students may appropriately infer that the *physical* capacitance remains the same, implying that the charge held is also doubled, thus *mathematically facilitating* their gaining *deeper physical understanding*.

The workshop will conclude with a reflective discussion upon the answers that in-service physicists gave to the same questions to further elucidate the (de)mathematisation phenomenology within the school unit. Such a process may facilitate the construction of meaningful teaching-learning bridges across the two courses, which importantly will help in gaining deeper understanding of both the taught content of the two distinct courses *and* their noematic convergences/divergences. We posit that through such interdisciplinary linkings, a novel quality of *interdisciplinary meta-learning* emerges, within which each discipline is viewed as a way of experiencing a phenomenon, thus highlighting both the distinctness of the disciplines and their convergences. Hence, the interdisciplinary linkings allow the conatruction of a meaningful communication space between mathematics and physics, thus facilitating appropriate "didactical planification towards to a meaningful learning as linking links" (Moutsios-Rentzos & Kalavasis, 2016, p. 97). We argue that such a quality of learning seems to be appropriate for the contemporary integrated, virtual society.

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